Q1-(a)

$y=t^2$ X= t and

$z^2=4-t^2-t^4$

$t^2, \sqrt{4-t^2-t^4}$ ¿ , So, r(t)= (t

(b)

$z^2=9+16=25, z=5.$, X=3 , y= 4

.So the point is (3,4,5)

$\nabla . f=(2x)i+(2y)j-(2z)k$

¿ $6i+8j-10k=$¿ *the normal vecto.*

The tangent plane is given by

$6(x-3)+8(y-4)-10(z-5)=0$

$6x-18+8y-32-10z+50=0$

$6x+8y-10z=0$

. Then we get the normal line question

$(x-3)i+(y-4)j+(z-5)=t(6i+8j-10k)$

$6t=x-3=\gg x=3+6t$

$8t=y-4=\gg y=4+8t$

$-10t=z-5=\gg z=5-10t$

↑ The normal line

(c)

.f(x,y,z) = xyz , V = i - 2j + k and W = 2i + j – k

$\nabla grad\ \frac{a}{a}$

i j k

1 2- 1

1- 1 2

V = i (2 – 1) – j (-1 – 2) + k (1 + 4)

i + 3j + 5k =

$\nabla=(yz)i+2j-k$

2i + 2j – k- =

$\frac{(-2i+2j-k)(i+3j+5k)}{\sqrt{35}}$

¿ $\frac{-2+6-5}{\sqrt{35}}=\frac{-1}{\sqrt{35}}$

(d)

e^{-y}¿ kF=($z^2 e^{-x}$) i + ($y^3 \ln z$)j + (x

:Solution

$\frac{x e^{-y}}{\partial z}$ + div . V ¿ $\frac{z^2 e^{-x}}{\partial x}$i + $\frac{y^3 \ln z}{\partial y}$j

¿$-e^{-x} z^2 + 3 y^2 \ln z$ $\nabla . V = -z^2 e^{-x} + 3 y^2 \ln z + 0$

:For the curl

i	j	k
$\frac{d}{dz}$	$\frac{d}{dy}$	$\frac{d}{dx}$
e^{-y}x	$y^3 \ln z$	$z^2 e^{-x}$

($\frac{dz^2 e^{-x}}{dy}$- $\frac{dy^3 \ln z}{dx}$)Curl = i($\frac{dx e^{-y}}{dy}$ - $\frac{dy^3 \ln z}{dz}$) – j($\frac{dx e^{-y}}{dx}$ - $\frac{dz^2 e^{-x}}{dz}$) + k

¿$\left(-x e^{-y} - \frac{y^3}{z}\right) i - \left(e^{-y} - 2 z e^{-x}\right) j + (0 - 0) k$

.¿$\left(-x e^{-y} - \frac{y^3}{z}\right) i - \left(e^{-y} - 2 z e^{-x}\right) j$

Q2- evaluate the following integrations

(a)

$$\int_0^1 \int_0^1 x^2 e^{xy}\, dy dx$$

$$\int_0^1 \frac{x^2 e^{xy}}{x} \Big|_0^1 dx$$

$$\int_0^1 x(¿\, e^x - 1)\, dx¿$$

$$\int_0^1 x e^x - x\, dx$$

$$\Big|_0^1 (\; ¿¿$$

$$¿ (e - e - \frac{1}{2}) - (0 - 1 - 0)$$

$$¿ 1 - \frac{1}{2} = \frac{1}{2}.$$

Q2- evaluate the following integrations

$$\iint_{0\,0}^{2\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\,dy\,dx$$

$$\int_0^2 ¿¿$$

$$\frac{1}{3}\int_0^2 \left[x^2+\left(2x-x^2\right)\right]\frac{3}{2}-\left(x^2\right)\frac{3}{2}\,dx$$

$$\frac{1}{3}\int_0^2 ¿¿$$

$$\frac{1}{3}\int_0^2 \sqrt{8}\,x^{\frac{3}{2}}-x^3\,dx$$

$$¿\frac{1}{3}\left(\sqrt{8}.\frac{2}{5}.x^{\frac{5}{2}}-\frac{x^4}{4}\right)\Big|_0^2$$

$$¿\frac{1}{3}\left(\sqrt{8}\sqrt{32}.\frac{2}{5}-4\right)=\frac{4}{5}$$

.

(c)

$$\iint_{\frac{-\pi}{3}\,1+\cos\theta}^{\frac{\pi}{3}\,3\cos\theta} r\,dr\,d\theta$$

$$\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{r^2}{2}\frac{3\cos\theta}{1+\cos\theta}d\theta=\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \text{¿¿¿}$$

$$\text{¿}\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} 9\cos^2\theta-\left(1+2\cos\theta+\cos^2\theta\right)d\theta=\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} 9\cos^2\theta-\left(1-2\cos\theta-\cos^2\theta\right)d\theta$$

$$\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} 8\cos^2\theta-2\cos\theta-1\,d\theta=\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{8}{2}(1+\cos 2\theta)-2\cos\theta-1\,d\theta$$

$$\frac{1}{2}\left(\frac{8}{2}\theta+\frac{8\sin 2\theta}{4}-2\sin\theta-\theta\right)\Bigg|_{\frac{-\pi}{3}}^{\frac{\pi}{3}}=\frac{1}{2}\left(\frac{8\pi}{6}+8\frac{\sqrt{3}}{8}-2\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)$$

$$\text{¿}\frac{1}{2}\left(\frac{8\pi}{6}+\sqrt{3}-\sqrt{3}-\frac{\pi}{3}\right)=\left(\frac{8\pi}{12}-\frac{\pi}{6}\right)-\frac{1}{2}\left(\frac{-8}{6}-\frac{8\sqrt{3}}{8}+\sqrt{3}+\frac{\pi}{3}\right)$$

$$\text{¿}\left(\frac{8\pi}{12}-\frac{\pi}{6}\right)+\left(\frac{8}{12}-\frac{\pi}{6}\right)=\frac{16\pi}{12}-\frac{2\pi}{6}$$

$$\text{¿}\frac{8\pi}{6}-\frac{2\pi}{6}=\frac{6\pi}{6}=\pi$$

(d)

$0 \le x \le 3, 0 \le y \le 6, 0 \le z \le 2$

, So the value is given by

$$\int_0^3 \int_0^6 \int_0^2 dz\, dy\, dx$$

$$\int_0^3 \int_0^6 [z]_0^2 \, dy\, dx$$

$$\int_0^3 [2y]_0^6 \, dx$$

$$\int_0^3 12 \, dx$$

$$[12x]_0^3 = 36$$

Q-3

$x=t, y=t^2+2t, z=0.$

$r(t)=[t, t^2, 2t]$

$r'(t)=[1, 2t, 2]$

$F[r(t)]=[t^2+2t, t^2+3t]$

$Fr(t).r'(t)=t^2+2t+2t^3+8t^2+6t$

The line integral is given by

$$\int_0^2 t^2+2t+2t^3+8t^2+6t\,dt$$

$$\int_0^2 9t^2+2t^3+8t\,dt$$

$$\left[3t^3+\frac{t^4}{2}+4t^2\right]_0^2=24+8+16=48$$

Q4- Green's theorem

$$\oint_c ((x-y^2¿)\,dx+(1+x^2)\,dy)¿$$

$$y^2=x \wedge y=x^3$$

$$0 \le x \le 1$$

:Solution

: First

i	j	k		
$\frac{d}{dz}$	$\frac{d}{dy}$	$\frac{d}{dx}$		
0	$1+x^2$	$x-y^2$		

$$(\frac{x-y^2}{dy}- \frac{1+x^2}{dx})i(0 - \frac{1+x^2}{dz}) + j(0 - \frac{x-y^2}{dz}) + k =$$

$$¿(2x+2y)k$$

$$\int_0^1 \int_{\sqrt{x}}^{x^3} 2x+2y\,dy\,dx=\int_0^1 2xy+y^2 \Big|_{\sqrt{x}}^{x^3} dx$$

$$\frac{x^2}{2}\Big|_0^1 - \int_0^1 2x^4+x^3-x^{\frac{1}{2}}-x\,dx=\frac{2x^5}{5}+\frac{x^4}{4}-\frac{2x^{\frac{5}{2}}}{5}$$

$$\frac{2}{5}+\frac{1}{4}-\frac{2}{5}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}=\frac{-1}{4}$$

Q – 5

$$\iint_s F.n\,dA$$

$y^2=1+x^2$ The cone . $z^2=x^2+y^2$ with cylinder

$F=2i+5j+3k$

$\sqrt{x^2+y^2}\ when\ 0\le z\le 1$ =Since z

Parametric representation of cone

$r(r,\theta)[rcos\theta, rsin\theta, r]$

$ru\times rv$ =N

$N=r(r)\times r(\theta)$

$r(r)=[cos\theta, sin\theta, 1]$

$r(\theta)=[-rsin\theta, rcos\theta, 0]$

$r(r)\times r(\theta)=$¿

$$\begin{vmatrix} i & j & k \\ cos\theta & sin\theta & 1 \\ -rsin\theta & rcos\theta & 0 \end{vmatrix}$$

$-rcos\theta i-rsin\theta j+rk$

$Fr(r,\theta)=[2,5,3]$

$F.N=-2rcos\theta-5rsin\theta+3r$

$0\le r\le 1$

$0\le\theta\le 2\pi$

$$\int_0^{2\pi}\int_0^1 -2rcos\theta-2rsin+3r\,drd\theta$$

$$\int_0^{2\pi} -r^2\cos\theta - \frac{5}{2}r^2\sin\theta + \frac{3}{2}$$

$$\left[-\sin\theta + \frac{5}{2}\cos\theta + \frac{3}{2}\theta\right]_0^{2\pi}$$

$$\frac{5}{2} + 3\pi - \frac{5}{2} = 3\pi$$

Q – 6

$F = [z, x, y]$

Hince

$z = \sqrt{x^2 + y^2}, 0 \le z \le 4$

¿ $z-\sqrt{x^2+y^2}$ g(x,y,z)=z – f(x,y)=0

$\left[\frac{-x}{\sqrt{x^2+y^2}},\frac{-y}{\sqrt{x^2+y^2}},1\right]$ =grad g

=N

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ z & x & y \end{vmatrix}$$

i + j + k =

$\frac{-x}{\sqrt{x^2+y^2}}+\frac{-y}{\sqrt{x^2+y^2}}+1$=grad g. N

by using polar form

$0 \le r \le 4$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^4 \frac{-x}{\sqrt{x^2+y^2}}+\frac{-y}{\sqrt{x^2+y^2}}+1\,dA$$

$$\int_0^{2\pi}\int_0^4 \left(\frac{-r\cos\theta}{r}+\frac{-r\sin\theta}{r}+1\right) r\,dr\,d\theta$$

$$\int_0^{2\pi} (-\cos\theta-\sin\theta+1)\frac{r^2}{2}\Big|_0^4 d\theta$$

$$8(-\sin\theta+\cos\theta+\theta)\Big|_0^{2\pi}$$

¿ 16π.

-Q7

div. V= 2x + 1 + 2z

$$\int_0^{2\pi}\int_0^{2}\int_0^{2-r\cos\theta} 2r\cos\theta+1+2z\,dz\,r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 2r^2\cos\theta z+rz+rz^2\Big|_0^{2-r\cos\theta} dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 2r^2\cos\theta(2-r\cos\theta)+r(2-r\cos\theta)+r(2-r\cos\theta)^2\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 4r^2\cos\theta-2r^3\cos^2\theta+2r-r^2\cos\theta+4r-4r^2\cos\theta+r^2\cos^2\theta\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 6r-2r^3\cos^2\theta-r^2\cos\theta+r^2\cos^2\theta\,dr\,d\theta$$

$$\int_0^{2\pi} 3r^2-\frac{r^4}{2}\cos^2\theta-\frac{r^3}{3}\cos\theta+\frac{r^3}{3}\cos^2\theta\Big|_0^2\,d\theta$$

$$\int_0^{2\pi} 12-8\cos^2\theta-\frac{8}{3}\cos\theta+\frac{8}{3}\cos^2\theta\,d\theta$$

$$\int_0^{2\pi} 12-4(1+\cos 2\theta)-\frac{8}{3}\cos\theta+\frac{4}{3}(1+\cos 2\theta)\,d\theta$$

$$\left(12-\frac{8}{3}\theta-\frac{4}{3}\sin 2\theta-\frac{8}{3}\sin\theta\right)\Big|_0^{2\pi}=\left(24\pi-\frac{16}{3}\pi-0-0\right)$$

¿ $\frac{56\pi}{3}$